RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2015-18]

B.A./B.Sc. SECOND SEMESTER (January – June) 2016

Mid-Semester Examination, March 2016

Date : 21/03/2016 — **MATH FOR ECO (General)**

Time : 12 noon – 1 pm — Paper : II — Full Marks : 25

Answer <u>any three</u> questions : [3×4]

1. a) Prove that, $\lim_{x\to a} f(x) = \ell$ if and only if for every sequence $\{a_n\}$ converging to a; $f(a_n)$ converges to ℓ. [2]

   b) Is $\lim_{x\to 0} \sin\frac{1}{x}$ exists. Justify your answer. [2]

2. a) Prove that $\lim_{x\to 0} \frac{\sin x}{x} = 1$. [3]

   b) Find out the limit, $\lim_{x\to 0} \frac{1-\cos x}{x}$. [1]

3. a) Prove that f(x) = cosx, $x \in \mathbb{R}$; is continuous on $\mathbb{R}$. [2]

   b) Use intermediate value theorem to show that, every odd degree polynomial with real co-efficients; must have a real solution. [2]

4. a) Let $f : (a,b) \to \mathbb{R}$ be a function and let $c \in (a,b)$. Also assume that f is differentiable at c. Then show that f is continuous at c. [2]

   b) Prove that $\frac{d}{dx}(\sin x) = \cos x$. [2]

5. a) Calculate the limit, $\lim_{x\to\infty} \frac{\sin x}{x + \cos x}$. [2]

   b) Find $\frac{dy}{dx}$, where $y = x^{\tan x} + (\sin x)^{\cos x} + 10x^2$. [2]

6. **Answer <u>any two</u> questions :** [2×5]

   a) Consider the vector space $\mathbb{R}_{2\times 2}$ of all real 2nd order matrices and S is the set of all 2×2 real skew symmetric matrices. Prove that S is a subspace of $\mathbb{R}_{2\times 2}$. [5]

   b) State the Replacement theorem. Use it to find a basis for the vector space $\mathbb{R}^3$, that contains the vectors (1,2,1) and (2,1,1). [2+3]

   c) Solve the system of equations : [5]

   $$x + 2y + z - 3\omega = 0$$
   $$2x + 4y + 3z + \omega = 0$$
   $$3x + 6y + 4z - 2\omega = 0$$

7. **Answer <u>any one</u> question :** [1×3]

   a) Let V be a real vector space with $\{\alpha, \beta, \gamma\}$ as a basis. Prove that the set $\{\alpha+\beta+\gamma, \beta+\gamma, \gamma\}$ is also a basis of V. [3]

   b) Justify that the union of two subspaces of a vector space may not be a subspace. [3]

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